

Fundamental Optics Principles

- Show your work. Write vergences and powers with a sign (+ or –) and units (D).
- Light travels left to right. Distances to the left of a lens are negative.
- Check your answers in the interactive worksheet on the course site.
- Convert distances to meters before taking a reciprocal.
- Use $n(\text{air}) = 1.000$ and $n(\text{water}) = 1.333$ unless told otherwise. $c = 3.00 \times 10^8$ m/s.
- Parts F and G are extensions. They introduce the Lensmaker's and Thin Lens equations using the vergence ideas from this lecture.

A Vergence & Collimation

$$V = \frac{1}{d(\text{m})} \quad \text{Diverging light: negative} \cdot \text{Converging light: positive} \cdot \text{Collimated: } 0 \text{ D}$$

- A1** A point source is 2 m from an observer. What vergence reaches the observer? (2 pts)

Answer: –0.50 D

Diverging light is negative: $V = -1/2 \text{ m} = \mathbf{-0.50 \text{ D}}$.

- A2** What is the vergence of light 25 cm from a point source? (2 pts)

Answer: –4.00 D

Convert first: 25 cm = 0.25 m. $V = -1/0.25 = \mathbf{-4.00 \text{ D}}$.

- A3** A lens focuses light to a point 50 cm away. What vergence does the light have as it leaves the lens? (2 pts)

Answer: +2.00 D

Converging light is positive: $V = +1/0.50 \text{ m} = \mathbf{+2.00 \text{ D}}$.

- A4** An observer measures –0.10 D of divergence from a point source. How far away is the source, in meters? (2 pts)

Answer: 10 m

$d = 1/|V| = 1/0.10 = \mathbf{10 \text{ m}}$.

- A5** Which observer receives light with the greatest magnitude of vergence? (1 pt)

A 10 cm from the source

B 1 m from the source

C 6 m from the source

D 100 m from the source

Answer: 10 cm from the source (–10.00 D)

Vergence grows as distance shrinks: $1/0.10 \text{ m} = 10 \text{ D}$ is the largest.

A6 Optical infinity is 6 m. What is the actual vergence of light from a point source 6 m away? (2 pts)

Answer: -0.17 D (treated as 0)

$V = -1/6 = -0.17$ D. That is small enough that clinically we treat it as collimated (0 D).

A7 Collimated light has a vergence of: (1 pt)

A 0 D

B +1.00 D

C -1.00 D

D Infinite

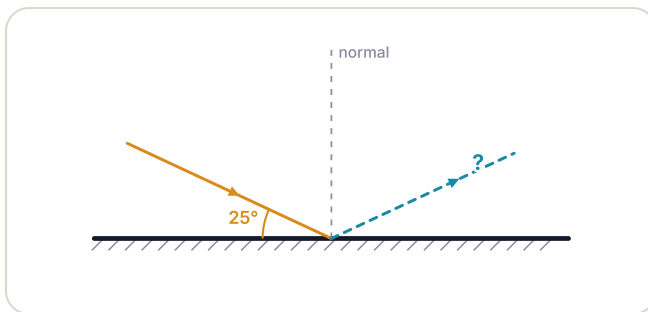
Answer: 0 D

Collimated rays are parallel, neither converging nor diverging, so $V = 0$ D.

B Reflection

$\theta_i = \theta_r$ Angles from the normal · angle from surface = 90° – angle from normal

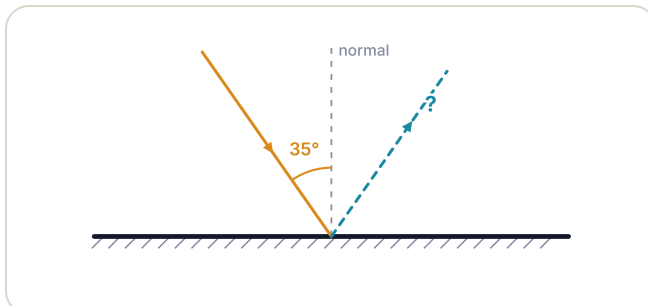
B1 A ray strikes a mirror 25° from the surface. At what angle does it leave, measured from the surface? (2 pts)



Answer: 25°

The law of reflection holds from the surface too: 25° .

B2 A ray strikes a mirror 35° from the normal. What angle does the reflected ray make with the mirror's surface? (2 pts)



Answer: 55°

It reflects at 35° from the normal, which is $90^\circ - 35^\circ = 55^\circ$ from the surface.

- B3** A ray strikes a mirror 30° from the normal. What is the angle between the incident ray and the reflected ray? (2 pts)

Answer: 60°

Each ray is 30° from the normal on opposite sides: $30^\circ + 30^\circ = 60^\circ$.

- B4** Challenge: a ray strikes a mirror 20° from the surface. What is the angle between the incident and reflected rays? (3 pts)

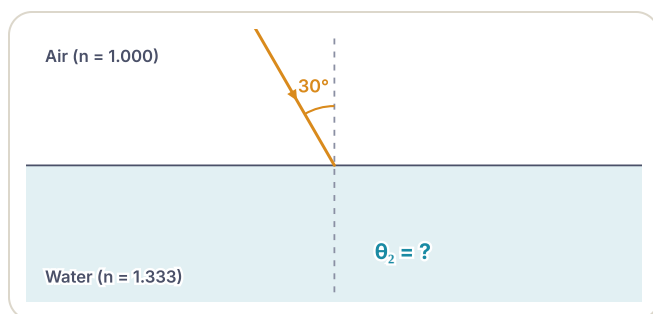
Answer: 140°

From the normal, the ray is $90^\circ - 20^\circ = 70^\circ$. Incident and reflected rays are each 70° from the normal: $70^\circ + 70^\circ = 140^\circ$.

C Refraction, Snell's Law & Index of Refraction

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad n = \frac{c}{v}$$

- C1** Light passes from air into water at 30° from the normal. What is the angle of refraction? (3 pts)



Answer: 22.0°

$\sin \theta_2 = (1.000 \times \sin 30^\circ) / 1.333 = 0.500 / 1.333 = 0.375$, so $\theta_2 = \sin^{-1}(0.375) = 22.0^\circ$.

- C2** Light passes from air into crown glass ($n = 1.523$) at 45° . What is the angle of refraction? (3 pts)

Answer: 27.7°

$\sin \theta_2 = 0.7071 / 1.523 = 0.4643$, so $\theta_2 = 27.7^\circ$.

- C3** How fast does light travel in water ($n = 1.333$)? Give your answer in units of 10^8 m/s. (2 pts)

Answer: 2.25×10^8 m/s

$v = c/n = 3.00 \times 10^8 / 1.333 = 2.25 \times 10^8$ m/s.

- C4** Light travels at 1.89×10^8 m/s in a lens material. What is its index of refraction, and which material is it? (2 pts)

Answer: 1.59 (polycarbonate)

$n = c/v = 3.00 / 1.89 = 1.59$, which is polycarbonate.

C5 When light passes from air into water, it bends:

(1 pt)

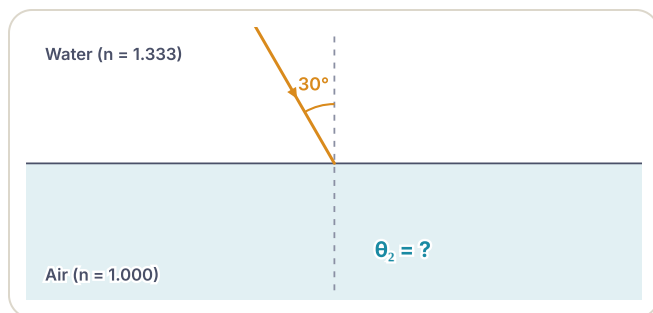
- ☒ **A Toward the normal** ☐ **B Away from the normal**
☐ **C It does not bend** ☐ **D It all reflects**

Answer: Toward the normal

Entering a higher index, light slows and bends toward the normal.

C6 Light leaves water and enters air, striking the surface at 30° from the normal. What is the angle of refraction in air?

(3 pts)



Answer: 41.8°

$\sin \theta_2 = 1.333 \times \sin 30^\circ / 1.000 = 0.6665$, so $\theta_2 = \mathbf{41.8^\circ}$. Light bends away from the normal entering a lower index.

C7 A ray in air hits an unknown material at 40° and refracts to 25° . What is the material's index?

(3 pts)

Answer: 1.52

$n_2 = \sin 40^\circ / \sin 25^\circ = 0.6428/0.4226 = \mathbf{1.52}$ (crown glass).

C8 Challenge: beyond what angle in water (from the normal) does light fail to escape into air at all?

(3 pts)

Answer: 48.6° (critical angle)

The refracted angle reaches 90° when $\sin \theta_1 = 1.000/1.333 = 0.750$, so $\theta = \mathbf{48.6^\circ}$. Beyond this *critical angle* the light is totally internally reflected.

D Lenses, Focal Points & the Vergence Equation

$$P = \frac{1}{f(\text{m})} \quad V = U + P \quad U = \text{vergence entering the lens} \cdot V = \text{vergence leaving}$$

- D1** A lens converges collimated light to a point 20 cm behind it. What is its power? (2 pts)

Answer: +5.00 D

$$P = 1/0.20 \text{ m} = \mathbf{+5.00 \text{ D.}}$$

- D2** After passing through a lens, collimated light appears to diverge from a point 50 cm in front of it. What is the lens power? (2 pts)

Answer: -2.00 D

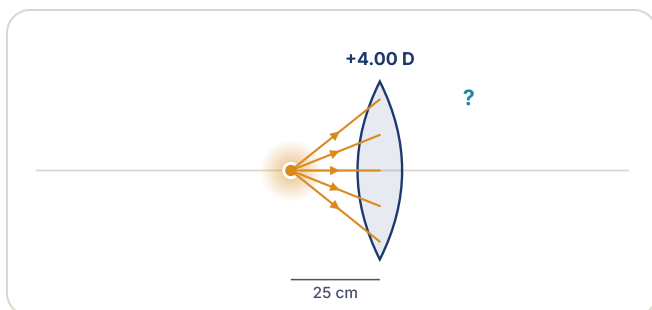
$$\text{The focal point is to the left: } P = 1/-0.50 \text{ m} = \mathbf{-2.00 \text{ D.}}$$

- D3** What is the focal length of a +8.00 D lens, in centimeters? (2 pts)

Answer: 12.5 cm

$$f = 1/8 = 0.125 \text{ m} = \mathbf{12.5 \text{ cm.}}$$

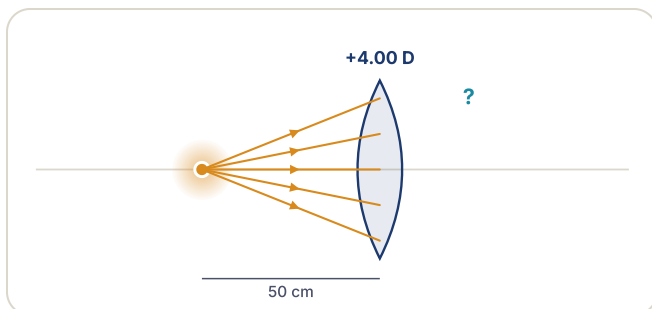
- D4** A point source sits 25 cm to the left of a +4.00 D lens. What vergence leaves the lens? (3 pts)



Answer: 0.00 D (collimated)

$U = -1/0.25 = -4.00 \text{ D}$. $V = U + P = -4.00 + 4.00 = \mathbf{0.00 \text{ D}}$. The source is at the front focal point, so the light leaves collimated.

- D5** The same source is moved to 50 cm left of the +4.00 D lens. What vergence leaves the lens? (3 pts)



Answer: +2.00 D

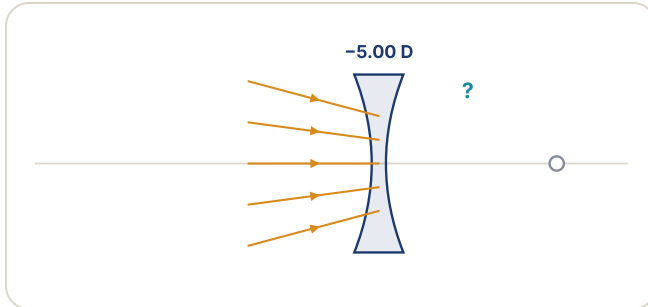
$$U = -1/0.50 = -2.00 \text{ D}$$
. $V = -2.00 + 4.00 = \mathbf{+2.00 \text{ D}}$ (converging).

- D6** For question D5, how far to the right of the lens does the light come to a focus, in centimeters? (2 pts)

Answer: 50 cm

$$d = 1/V = 1/2.00 = 0.50 \text{ m} = \mathbf{50 \text{ cm.}}$$

- D7** Converging light of +2.00 D strikes a -5.00 D lens. What vergence leaves the lens, and is it converging or diverging? (3 pts)



Answer: -3.00 D (diverging)

$$V = +2.00 + (-5.00) = \mathbf{-3.00 \text{ D}}, \text{ so the light is diverging.}$$

- D8** Using this lecture's convention, the focal point where collimated light seems to originate for a negative lens is: (1 pt)

- ☐ A Real, to the right of the lens
☐ B **Virtual, to the left of the lens**
☐ C Real, to the left of the lens
☐ D At the center of the lens

Answer: Virtual, to the left

Diverged light only appears to come from a point on the left. No light actually goes there, so it is virtual.

E Applications & Concepts

- E1** Why can a high-index lens be thinner than a CR-39 lens of the same prescription? (1 pt)

- ☐ A **It bends light more per unit of curvature, so less curvature (and material) is needed**
☐ B It reflects more light
☐ C It slows light down less
☐ D It has a higher Abbe value

Answer: Bends light more per unit curvature

A higher index bends light more strongly, so flatter curves reach the same power.

- E2** A spear-fisher sees a fish ahead and below. To hit it they should aim: (1 pt)

- ☐ A At the image
☐ B Beyond the image
☐ C **Below the image**
☐ D Above the image

Answer: Below the image

Refraction at the surface makes the fish look shallower than it is, so aim below the image.

E3 In a primary rainbow, which color appears on the outside (top) of the bow? (1 pt)

- ☐ A Violet ☐ B Green
☒ C Red ☐ D Blue

Answer: Red

Red exits the drop at $\approx 42^\circ$ and violet at $\approx 40^\circ$ from the antisolar point, so red is on the outside.

E4 Which lens material is the usual choice for a child's safety glasses? (1 pt)

- ☐ A Crown glass ☐ B CR-39
☒ C Polycarbonate ☐ D High-index 1.74

Answer: Polycarbonate

Polycarbonate (or Trivex) is highly impact-resistant and lightweight.

E5 True or false: a lens with a focal length of 2 m is stronger than a lens with a focal length of 0.5 m. (1 pt)

- ☐ A True ☒ B False

Answer: False

False. Power = $1/f$: $1/2 = 0.50$ D versus $1/0.5 = 2.00$ D. A shorter focal length means a stronger lens.

E6 In two or three sentences, explain why a mirage appears on a hot road. (3 pts)

Answer: Hot air has lower n ; light curves up toward the eye, showing sky on the road.

Air near the hot road is warmer and has a slightly lower index of refraction than the cooler air above. Light from the sky traveling down toward the road bends gradually upward through these layers and reaches your eye from below, so you see an image of the sky on the road that looks like water.

F Extension: The Lensmaker's Equation

A lens's power depends on its material (n) and the curvature of its two surfaces. For a thin lens in air:

$$P = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

R_1 is the front surface radius and R_2 the back surface radius, **in meters**. A radius is positive when its center of curvature lies to the *right* of the surface, and negative when it lies to the left. A flat surface has $R = \infty$ (so $1/R = 0$).

- F1** A biconvex lens has $n = 1.50$, $R_1 = +20$ cm and $R_2 = -20$ cm. What is its power? (3 pts)

Answer: +5.00 D

$P = 0.50 \times (1/0.20 - 1/-0.20) = 0.50 \times (5 + 5) = +5.00$ D.

- F2** The same shape is made in 1.74 high-index plastic. What is its power now? (3 pts)

Answer: +7.40 D

$P = 0.74 \times 10 = +7.40$ D. The same curves give more power, which is why high-index lenses can use flatter, thinner curves for a given prescription.

- F3** A plano-concave polycarbonate lens ($n = 1.586$) has a flat front ($R_1 = \infty$) and a back surface with $R_2 = +10$ cm. What is its power? (3 pts)

Answer: -5.86 D

$P = 0.586 \times (0 - 1/0.10) = 0.586 \times (-10) = -5.86$ D.

- F4** Design problem: what front radius (in cm) gives a plano-convex crown glass lens ($n = 1.523$, flat back) a power of +4.00 D? (3 pts)

Answer: $R_1 = +13.1$ cm

With $1/R_2 = 0$: $R_1 = (n - 1)/P = 0.523/4.00 = 0.131$ m = **13.1 cm**.

G Extension: The Thin Lens Equation

The thin lens equation is the vergence equation written with distances. The object distance u is negative for a real object to the left of the lens; a positive image distance v means a real image to the right.

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} \iff V = U + P \quad m = \frac{v}{u}$$

Magnification m : negative means inverted, positive means upright; $|m| > 1$ means enlarged.

- G1** An object is 40 cm to the left of a +5.00 D lens. Where is the image? (Signed distance in cm.) (3 pts)

Answer: +40 cm

$U = 1/-0.40 = -2.50$ D. $V = -2.50 + 5.00 = +2.50$ D. $v = 1/2.50 = 0.40$ m = **+40 cm** (real image, right of the lens).

- G2** What is the magnification of the image in G1? (2 pts)

Answer: -1 (inverted, same size)

$m = v/u = 40/-40 = -1$: inverted and the same size.

- G3** The object moves to 10 cm left of the same +5.00 D lens. Where is the image now? (Signed distance in cm.) (3 pts)

Answer: -20 cm (virtual, m = +2)

$U = 1/-0.10 = -10.00$ D. $V = -10 + 5 = -5.00$ D. $v = 1/-5 = -0.20$ m = **-20 cm**: a virtual, upright image on the left. $m = -20/-10 = +2$, which is how a magnifier works.

- G4** An object is 50 cm to the left of a -4.00 D lens. Where is the image? (Signed distance in cm.) (3 pts)

Answer: -16.7 cm

$U = -2.00$ D. $V = -2.00 - 4.00 = -6.00$ D. $v = 1/-6 = -16.7$ cm (virtual).

- G5** Challenge: where must an object be placed so a +10.00 D lens forms a real image 30 cm to its right? (Signed distance in cm.) (3 pts)

Answer: -15 cm

$V = 1/0.30 = +3.33$ D. $U = V - P = 3.33 - 10.00 = -6.67$ D. $u = 1/-6.67 = -15$ cm (15 cm left of the lens).

Numeric answers accept small rounding differences. Award partial credit for correct setup with arithmetic errors.